

## A STRENGTHENED CAUCHY–SCHWARZ INEQUALITY FOR BIDIMENSIONAL SPLINE–WAVELETS

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*Abstract.* Multilevel methods have been widely applied for the approximation of the solutions of the elliptic partial differential equations in the frame of finite element spaces and, recently, owing to the development of the wavelet theory, in wavelet spaces. It has been pointed out that the strengthened Cauchy-Buniakowski-Schwarz inequality is the main tool in the analysis of multilevel methods. In this paper, results on the strengthened Cauchy-Buniakowski-Schwarz inequality are reviewed for one dimensional biorthogonal wavelets and, as original contribution, a theorem is proposed for the bidimensional case, concerning order one spline-wavelets.

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