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MULTIVARIABLE MIXED MEANS AND INEQUALITIES OF HARDY AND LEVIN–COCHRAN–LEE TYPE

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Abstract. We consider integral power means of arbitrary real order, taken over cells in $\mathbf{R}^n$, and their dual means. We establish related mixed-means inequalities and then apply obtained results to derive multivariable analogues and some new generalizations of Hardy and Levin-Cochran-Lee type inequalities. Moreover, we prove the constant factors involved in the right-hand sides of these relations to be the best possible, that is, they cannot be replaced with smaller constants.

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