

## GENERALISED TRAPEZOIDAL RULES WITH ERROR INVOLVING BOUNDS OF THE $n$ TH DERIVATIVE

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*Abstract.* Inequalities obtained for the generalised trapezoidal rules in terms of the upper and lower bounds of the  $n$ th derivative of the integrand. The Hayashi inequality, or more appropriately an inequality due to Steffensen, is utilised to procure the results which contain earlier expressions as particular cases.

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