

CHOQUET THEORY FOR SIGNED MEASURES

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Abstract. We introduce the notion of barycenter for a class of non-necessarily positive Radon measures and prove on this basis several inequalities which extend classical results such as Steffensen's inequality, Fink's version of the Hermite-Hadamard inequality, Fuchs' extension of the majorization inequality of Hardy-Littlewood-Polya etc.

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