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INEQUALITIES FOR THE GAMMA FUNCTION RELATING TO ASYMPTOTIC EXPANSIONS

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Abstract. Many inequalities for the gamma function can be deduced from monotonicity or convexity properties of $\log\Gamma(x)$ and related functions involving finite sums of the Stirling asymptotic series. Considering a particular case of a generalization of this classical expansion, we deduce further convexity results and inequalities which are similar to some other ones related to the usual form of the Stirling series. We give, among other things, inequalities which overvalue $\log\Gamma(x)$, whereas the corresponding finite sums of the classical expansion undervalue it or vice versa. Moreover we obtain bilateral inequalities also for the digamma and the polygamma functions. Finally, a few extensions of Gautschi-type inequalities are discussed.

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