

## EXPONENTIAL STABILITY AND UNSTABILITY OF SEMIGROUPS OF LINEAR OPERATORS IN BANACH SPACES

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*Abstract.* Necessary and sufficient conditions for uniform exponential stability and uniform exponential instability of semigroups of linear operators are given, in terms of Banach function spaces and Banach sequence spaces, respectively. We generalize some well-known theorems proved by Datko, Pazy, Rolewicz and Neerven concerning the exponential stability of  $C_0$ -semigroups. We shall obtain the versions of the theorems mentioned above, for the case of uniform exponential instability of  $C_0$ -semigroups.

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*Key words and phrases:* Semigroup, uniform exponential stability, uniform exponential instability, Banach function spaces, Banach sequence spaces.

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