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A GENERAL FORM PRESERVING THE OPERATOR ORDER FOR CONVEX FUNCTIONS

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Abstract. We show a general form preserving the operator order for convex functions based on the Mond-Pečarić method as follows: Let A and B be positive operators on a Hilbert space H satisfying $M1_H \geqslant B \geqslant m1_H > 0$. Let $f(t)$ be a continuous convex function on $[m,M]$. If $g(t)$ is a continuous increasing convex function on $[m,M] \cup \mathrm{Sp}(A)$, then for a given $\alpha > 0$

$$A \geqslant B \geqslant 0 \quad \text{implies} \quad \alpha g(A) + \beta I_H \geqslant f(B)$$

where $\beta = \max_{m \leqslant t \leqslant M}\{f(m) + [(f(M) - f(m))/(M-m)](t-m) - \alpha g(t)\}$. We extend Kantorovich type operator inequalities via the Ky Fan-Furuta constant as applications. Among others, we show the following inequality: If $A \geqslant B > 0$ and $M1_H \geqslant B \geqslant m1_H > 0$, then

$$\frac{M^{p-1}}{m^{q-1}}A^q \geqslant \frac{(q-1)^{q-1}}{q^q}\frac{(M^p-m^p)^q}{(M-m)(mM^p-Mm^p)^{q-1}}A^q \geqslant B^p$$

holds for all $p > 1$ and $q > 1$.

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