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MULTILINEAR DIRECT AND REVERSE STOLARSKY INEQUALITIES

IOSIF PINELIS

Abstract. For any nonnegative measurable function $f:[0,1]\to\mathbb{R}$ and any $a>0$, let $Q(f,a)$ denote the Stolarsky transform of f, equal to $\int_0^1 f\left(x^{1/a}\right)dx$. Let S_n stand for the set of all permutations of the set $\{1,\dots,n\}$. It is shown that the function

$$(0,\infty)^n \ni \mathbf{a}=(a_1,\dots,a_n)\longmapsto \mathcal{Q}(\mathbf{a}):=\sum_{\sigma\in S_n}\prod_{i=1}^n Q\left(f_{\sigma(i)},a_i\right)$$

is Schur-convex if the functions $f_1,\dots,f_n$ are nonnegative and nondecreasing and Schur-concave if $f_1,\dots,f_n$ are nonnegative and nonincreasing. Necessary and sufficient conditions for the strict Schur convexity and concavity are given.

Similar results are obtained for certain "direct" and "reverse" extensions of the Stolarsky transform to measures.

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