

ON THE FEKETE–SZEGÖ AND ARGUMENT INEQUALITIES FOR STRONGLY CLOSE-TO-STAR FUNCTIONS

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Abstract. Let $\mathcal{CS}(\beta)$ be the class of normalized strongly close-to-star functions of order β in the open unit disk. We obtain sharp Fekete-Szegő inequalities for functions belonging to the class $\mathcal{CS}(\beta)$. Some sufficient conditions for close-to-star functions also are investigated in a sector. Furthermore, we consider the integral preserving properties for functions in $\mathcal{CS}(\beta)$.

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REFERENCES

- [1] O. P. AHUJA AND M. L. MOGRA, *Effect of second coefficients on close-to-convex and close-to-star functions*, Rend. Mat. (7) **4** (1985), 21–37.
- [2] H. R. ABDEL-GAWAD AND D. K. THOMAS, *The Fekete-Szegő problem for strongly close-to-convex functions*, Proc. Amer. Math. Soc. **114** (1992), 345–349.
- [3] C. CARATHÉODORY, *Über den variabilitätsbereich der fourier’schen konstanten von positiven harmonischen funktionen*, Rend. Circ. Mat. Palermo **32** (1911), 193–217.
- [4] M. FEKETE AND G. SZEGÖ, *Eine Bemerkung über ungerade schlichte funktion*, J. London Math. Soc. **8** (1933), 85–89.
- [5] W. JANOWSKI, *Some extremal problems for certain families of analytic functions*, Bull. Acad. Polon. Sci. Sér. Sci. Phys. Astronom. **21** (1973), 17–25.
- [6] F. R. KEOGH AND E. P. MERKES, *A coefficient inequality for certain classes of analytic functions*, Proc. Amer. Math. Soc. **20** (1969), 8–12.
- [7] W. KOEPF, *On the Fekete-Szegő problem for close-to-convex functions*, Arch. Math. (Basel) **49** (1987), 420–433.
- [8] R. R. LONDON, *Fekete-Szegő inequalities for close-to-convex functions*, Proc. Amer. Math. Soc. **117** (1993), 947–950.
- [9] S. S. MILLER AND P. T. MOCANU, *Differential subordinations and univalent functions*, Michigan Math. J. **28** (1981), 157–171.
- [10] S. S. MILLER AND P. T. MOCANU, *Differential Subordinations*, Marcel Dekker Inc. New York and Basel, 1999.
- [11] M. NUNOKAWA, *On the order of strongly starlikeness of strongly convex functions*, Proc. Japan Acad. Ser. A Math. Sci. **69** (1993), 234–237.
- [12] K. S. PADMANABHAN AND R. PARVATHAM, *On certain generalized close-to-star functions in the unit disc*, Ann. Polon. Math. **37** (1980), 1–11.
- [13] R. PARVATHAM AND S. SRINIVASAN, *On Pascu type α -close-to-star functions*, Publ. Inst. Math. (Beograd)(N. S.) **49(63)** (1991), 71–75.
- [14] A. PFLUGER, *On the functional $|a_3 - \lambda a_2^2|$ in the class S* , Complex Variables Theory Appl. **10** (1988), 83–95.
- [15] CH. POMMERENKE, *Univalent Functions*, Vandenhoeck and Ruprecht, Göttingen, 1975.
- [16] M. O. READE, *On close-to-convex univalent functions*, Michigan Math. J. **3** (1955–56), 59–62.

- [17] H. SILVERMAN AND E. M. SILVIA, *Subclasses of starlike functions subordinate to convex functions*, *Canad. J. Math.* **37** (1985), 48–61.
- [18] H. M. SRIVASTAVA AND S. OWA (EDITORS), *Current Topics in Analytic Function Theory*, World Scientific Publishing Company, Singapore, New Jersey, London, and Hong Kong, 1992.
- [19] H. M. SRIVASTAVA, A. K. MISHRA AND M. K. DAS, *The Fekete-Szegő problem for a subclass of close-to-convex functions*, *Complex Variables Theory Appl.* **44** (2001), 145–163.
- [20] T. V. SUDHARSAN, P. BALASUBRAHMANYAM AND K. G. SUBRAMANIAN, *On a subclass of close-to-star functions*, *J. Math. Phys. Sci.* **25** (1991), 343–350.