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ON A PROBLEM BY K. NIKODEM

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Abstract. Concerning a problem raised by K. Nikodem, we prove the following statement. If G is an Abelian group divisible by 2, H is a Hilbert space and ε is a nonnegative real number and a function $f : G \to H$ satisfies

$$\|f(x-y) - 2f(x) - 2f(y)\| \leqslant \|f(x+y)\| + \varepsilon \quad (x, y \in G),$$

then there exists a function $g : G \to H$ fulfilling

$$g(x+y) + g(x-y) - 2g(x) - 2g(y) = 0 \quad (x, y \in G)$$

and

$$\|f(x) - g(x)\| \leqslant \frac{5}{2}\varepsilon \quad (x \in G).$$

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REFERENCES

[1] J. ACZÉL, *The general solution of two functional equations by reduction to functions additive in two variables and with the aid of Hamel bases*, Glasnik Mat.-Fiz. Astronom. Ser. II, **20** (1965), 65–73.
[2] J. ACZÉL, J. DHOMBRES, Functional Equations in Several Variables, Cambridge University Press, Cambridge, 1989.
[3] A. GILÁNYI, *Eine zur Parallelogrammgleichung äquivalente Ungleichung*, Aequationes Math., **62** (2001), 303–309.
[4] D. H. HYERS, *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. USA **27** (1941), 222–224.
[5] J. L. W. V. JENSEN, *Om Fundamentalligningers Oplösning ven elementäre Midler*, Tidsskr. Mat. **(4) 2** (1878), 149–155.
[6] S. KUREPA, *On the quadratic functionals*, Publ. Inst. Math. Acad. Serbe Sci. Beograd **13** (1959), 57–72.
[7] S. KUREPA, *On P. Volkmann's paper*, Glas. Mat. Ser. III, **22(42)** (1987), 371–374.
[8] GY. MAKSA, *Remark on the talk of P. Volkmann*, Proc. of the Twenty-third International Symposium on Functional Equations (Gargnano, Italy, 1985), Centre for Information Theory, Faculty of Mathematics, Oniversity of Waterloo, Waterloo, Ontario, Canada, 72–73.
[9] GY. MAKSA, P. VOLKMANN, *Characterization of group homomorphisms having values in an inner product space*, Publ. Math. Debrecen **56** (2000), 197–200.
[10] K. NIKODEM, *7. Problem*, Report of Meeting, Aequationes Math., **61** (2001), 301.
[11] F. SKOF, *On some alternative quadratic equations*, Results Math. **27** (1995), 402–411.
[12] F. SKOF, *On some alternative quadratic equations in inner-product spaces*, Atti Sem. Mat. Fis. Univ. Modena **46** (1998), 951–962.
[13] JACEK TABOR, JÓZEF TABOR, *Solution of the 7. Problem posed by K. Nikodem*, Report of Meeting, Aequationes Math., **61** (2001), 307–309.
[14] P. VOLKMANN, *Pour une fonction réelle f l'inéquation $|f(x)+f(y)| \leqslant |f(x+y)|$ et l'équation de Cauchy sont équivalentes*, Proc. of the Twenty-third International Symposium on Functional Equations (Gargnano, Italy, 1985), Centre for Information Theory, Faculty of Mathematics, University of Waterloo, Waterloo, Ontario, Canada, 43.

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