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EQUIVALENCE OF $\ell^{\{p_n\}}$ NORMS AND SHIFT OPERATORS

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Abstract. Given bounded mappings $p,q:\mathbb{Z}\to[1,\infty)$ (shortly $p=\{p_n\}$, $q=\{q_n\}$) we can consider Banach function spaces $\ell^{\{p_n\}}$ and $\ell^{\{q_n\}}$ with variable exponents. The necessary and sufficient condition to the p, q for the equivalence of norms in Banach spaces $\ell^{\{p_n\}}$ and $\ell^{\{q_n\}}$ is given. Moreover, considering shift operators S_k given by $(S_k a)_n = a_{n-k}$, $n\in\mathbb{Z}$, we prove that the norms $\|S_k\|_{\ell^{\{p_n\}}\to\ell^{\{p_n\}}}$, $k\in\mathbb{Z}$ are uniformly bounded with respect to k if and only if the norm in $\ell^{\{p_n\}}$ is equivalent to a norm of a classical ℓ^r with some constant exponent r.

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