

OPERATOR MONOTONE FUNCTIONS OF SEVERAL VARIABLES

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Abstract. We propose a notion of operator monotonicity for functions of several variables, which extends the well known notion of operator monotonicity for functions of only one variable. The notion is chosen such that a fundamental relationship between operator convexity and operator monotonicity for functions of one variable is extended also to functions of several variables.

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