A NEW CARLSON TYPE INEQUALITY

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Abstract. Consider a measure space $(X, d\xi)$ on which weight functions v, v_0 and v_1 are defined, and let $\theta \in (0,1)$ and $p, p_0, p_1 \in \mathbb{R}_+$. We investigate the three-weight Carlson type inequality

$$\|f\,v\|_{L^p(X,d\xi)} \leqslant A\|f\,v_0\|_{L^{p_0}(X,d\xi)}^{\theta}\|f\,v_1\|_{L^{p_1}(X,d\xi)}^{1-\theta}$$

to hold for some constant $A < \infty$ and all measurable functions f. A fairly general inequality of this type is proved. This result may be regarded as a generalization and unification of some other recent results of this type in the literature.

REFERENCES

[1] ANDRIANOV, F. I., *Multidimensional Analogues of Carlson's Inequality and its Generalizations*, Izv. Vyssh. Uchebn. Zaved. Mat. **1(56)** (1960), 3–7 (Russian).

[2] BARZA, S., *Weighted Multidimensional Integral Inequalities and Applications*, PhD Thesis, Department of Mathematics, Luleå University of Technology, Luleå (1999).

[3] BARZA, S., BURENKOV, V., PEČARIĆ, J., PERSSON, L.-E, *Sharp Multidimensional Multiplicative Inequalities for Weighted L_p Spaces with Homogeneous Weights*, Math. Ineq. Appl. **1** (1998), 53–67.

[4] BARZA, S., PEČARIĆ, J., PERSSON, L.-E., *Carlson Type Inequalities*, J. of Inequal. & Appl. **2** (1998), 121–135.

[5] BELLMAN, R., *An Integral Inequality*, Duke Math. J. **10** (1943) 547–550.

[6] BERGH, J., *An Optimal Reconstruction of Sampled Signals*, J. Math. Anal. Appl **115** (1986), 574–577.

[7] BERGH, J., LÖFSTRÖM, J., *Interpolation Spaces*, Springer-Verlag, Berlin-Heidelberg-New York (1976).

[8] BERTOLO, J. I., FERNANDEZ, D. L., *A Multidimensional Version of the Carlson Inequality*, J. Math. Anal. Appl. **100** (1984), 302–306.

[9] BEURLING, A., *Sur les intégrales de Fourier absolument convergentes et leur application à une transformation fonctionelle*, C. R. Neuvième Congrès Math. Scandinaves 1938, Helsinki (1939), 345–366.

[10] CARLSON, F., *Une inégalité*, Ark. Mat. Astr. Fysik **25B** (1934), 1–5.

[11] CATON, W. B., *A Class of Inequalities*, Duke Math. J., **6** (1940), 442–461.

[12] GABRIEL, R. M., *An Extension of an Inequality Due to Carlson*, J. London Math. Soc. **12** (1937), 130–132.

[13] GUSTAVSSON, J., PEETRE, J., *Interpolation of Orlicz Spaces*, Studia Math. **60** (1977), 33–59.

[14] HARDY, G. H., *A Note on Two Inequalities*, J. London Math. Soc. **11** (1936), 167–170.

[15] KAMALY, A., *Fritz Carlson's Inequality and its Application*, Math. Scand. **86** (2000), 100–108.

[16] KAMALY, A., *Type-Inequality Problems in Fourier Analysis*, PhD Thesis, Department of Electromagnetic Theory, Royal Institute of Technology, Stockholm (1998).

[17] KJELLBERG, B., *Ein Momentenproblem*, Ark. Mat. Astr. Fysik **29A** (1942), 1–33.

[18] KJELLBERG, B., *On Some Inequalities*, C. R. Dixième Congrès Math. Scandinaves 1946, Jul. Gjellerups Forlag, Copenhagen (1946), 333–340.

[19] KRUGLJAK, N. YA., MALIGRANDA, L., PERSSON, L.-E., *A Carlson Type Inequality with Blocks and Interpolation*, Studia Math. **104** (1993), 161–180.

[20] LARSSON, L., *A Weighted Carlson Type Inequality with Applications*, Licentiate Thesis, Department of Mathematics, Uppsala University, (2001).

[21] LEVIN, V. I., *Sharp Constants in Carlson Type Inequalities*, Dokl. Akad. Nauk, SSSR **59** (1948), 635–638 (Russian).

[22] LEVIN, V. I., *Some Inequalities between Series*, Mat. Sb. **45** (1938), 341–345 (Russian).

[23] MITRINOVIĆ, D. S., *Analytic Inequalities*, Springer-Verlag, Berlin-Heidelberg-New York, 1979.

[24] MITRINOVIĆ, D. S., PEČARIĆ, J., FINK, A. M., *Classical and New Inequalities in Analysis*, Kluwer Acad. Publ., Dordrecht-Boston-London, 1993.

[25] PEETRE, J., *Sur le nombre de paramètres dans la définition de certains espaces d'interpolation*, Ricerche Mat. **12** (1963), 248–261.

[26] PIGOLKIN, G. M., *Multidimensional Inequalities of Carlson Type and Related Inequalities*, Sibirsk. Mat. Ž. **21** (1970), 151–160 (Russian).

[27] YANG, G.-S., FANG, J.-C., *Some Generalizations of Carlson's Inequalities*, Indian J. Pure Appl. Math. **30**(10) (1999), 1031–1040.

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