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ON WEIGHTED WEAK TYPE MAXIMAL INEQUALITIES FOR MARTINGALES

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Abstract. Let Φ be a Young function and (u, v) a pair of weights on a probability space. We consider the inequality

$$\sup_{\lambda \in (0,\infty)} \Phi(\lambda)\,\mathbb{E}\Big[u : \{Mf > \lambda\}\Big] \leqslant \mathbb{E}\Big[\Phi\Big(C|f_\infty|\Big)v\Big]$$

for martingales $f = (f_n)_{n\in\mathbb{Z}_+}$, where $Mf = \sup_{n\in\mathbb{Z}_+}|f_n|$ and $f_\infty = \lim_n f_n$ a.s. We give some necessary and sufficient conditions for this inequality to hold, and extend Uchiyama's result.

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