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ERGODIC THEOREMS FOR DYNAMIC RANDOM WALKS

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Abstract. Given any measure-preserving dynamical system $(Y, \mathscr{A}, \mu, T)$ and $g \in L^p(\mu)$, we study convergence of the sequence $\left\{ \frac{1}{n} \sum_{k=1}^{n} g \circ T^{S_k},\ n \geqslant 1 \right\}$ where S_k is a dynamic $\mathbb{Z}^r$-valued random walk generated by another dynamical system, namely an irrational rotation on the d-dimensional torus. In this paper, Van der Corput's inequality and number theory are used for studying ergodic theorems and universally representative random sequences.

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