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# ERGODIC THEOREMS FOR DYNAMIC RANDOM WALKS

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*Abstract.* Given any measure-preserving dynamical system $(Y, \mathscr{A}, \mu, T)$ and $g \in L^p(\mu)$, we study convergence of the sequence $\left\{ \frac{1}{n} \sum_{k=1}^{n} g \circ T^{S_k},\ n \geqslant 1 \right\}$ where $S_k$ is a dynamic $\mathbb{Z}^r$-valued random walk generated by another dynamical system, namely an irrational rotation on the $d$-dimensional torus. In this paper, Van der Corput's inequality and number theory are used for studying ergodic theorems and universally representative random sequences.

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*Key words and phrases*: Ergodic Theorem, Van der Corput's Inequality, Random Walk, Continued Fractions, Denjoy-Koksma's inequality, Low Discrepancy sequences, Strong Sweeping out property.

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