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CALCULUS PROOFS OF SOME COMBINATORIAL INEQUALITIES

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Abstract. Using calculus we show how to prove some combinatorial inequalities of the type log-concavity or log-convexity. It is shown by this method that binomial coefficients and Stirling numbers of the first and second kinds are log-concave, and that Motzkin numbers and secondary structure numbers of rank 1 are log-convex. In fact, we prove via calculus a much stronger result that a natural continuous "patchwork" (i.e. corresponding dynamical systems) of Motzkin numbers and secondary structures recursions are increasing functions. We indicate how to prove asymptotically the log-convexity for general secondary structures. Our method also applies to show that sequences of values of some orthogonal polynomials, and in particular the sequence of central Delannoy numbers, are log-convex.

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