

SOME INEQUALITIES FOR GRAPHS DERIVABLE FROM HÖLDER'S INEQUALITY

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Abstract. Inequalities are derived as advertised in the title. For instance, it is shown that for a finite simple graph G , $4m \leq n(J_1 + t_1)$, where m is the number of edges of G , n is the number of vertices, J_1 is the average (over the edges) of the sizes of the “joint neighborhoods” of adjacent vertices, and t_1 is the average (over the edges) of the numbers of triangles in which the edges appear; with equality if and only if G is regular.

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