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MONOTONICITY OF SEQUENCES INVOLVING CONVEX AND CONCAVE FUNCTIONS

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Abstract. Let f be an increasing and convex (concave) function on $[0,1)$ and ϕ a positive increasing concave function on $[0,\infty)$ such that $\phi(0)=0$ and the sequence $\left\{\phi(i+1)\left(\frac{\phi(i+1)}{\phi(i)}-1\right)\right\}_{i\in\mathbb{N}}$ decreases $\left(\text{the sequence } \left\{\phi(i)\left(\frac{\phi(i)}{\phi(i+1)}-1\right)\right\}_{i\in\mathbb{N}} \text{ increases}\right)$. Then the sequence $\left\{\frac{1}{\phi(n)}\sum_{i=0}^{n-1} f\left(\frac{\phi(i)}{\phi(n)}\right)\right\}_{n\in\mathbb{N}}$ is increasing.

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