

## ON $p$ -QUASIHYPONORMAL OPERATORS AND QUASISIMILARITY

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*Abstract.* In this paper we show some structural properties of a  $p$ -quasihyponormal operator via Löwner-Heinz inequality and Hansen inequality. As important applications, it is shown that the normal parts of quasisimilar injective  $p$ -quasihyponormal operators are unitarily equivalent and quasisimilar injective  $p$ -quasihyponormal operators have same spectra and essential spectra.

*Mathematics subject classification (2000):* 47B20.

*Key words and phrases:*  $p$ -quasihyponormal operator, quasisimilarity.

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