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PLANAR PACKINGS AND MAPPINGS RELATED TO CERTAIN MINMAX PROBLEMS

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Abstract. For any integer $N \geqslant 1$, we obtain the extremal values of the minmax problem for exponential sums,

$$f(N) := \min_{a_i \text{ real}} \max \left\{ \left| \sum_{n=1}^{N} e^{ia_n} \right|, \left| \sum_{n=1}^{N} e^{iNa_n} \right| \right\}.$$

In particular, the two extremal problems $f(3)$ and

$$\max_{a,b,c \in [0,2\pi]} \min \left\{ \left| (e^{ia} - e^{ib})(e^{ib} - e^{ic})(e^{ic} - e^{ia}) \right|, \left| (e^{i3a} - e^{i3b})(e^{i3b} - e^{i3c})(e^{i3c} - e^{i3a}) \right| \right\}$$

are reduced to the problems about the packing of certain convex sets in the plane. This packing method also can be used to solve some other extremal problems.

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