

INEQUALITIES INVOLVING THE SEQUENCE $\sqrt[3]{a + \sqrt[3]{a + \cdots + \sqrt[3]{a}}}$

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Abstract. In this article, the convergence of the sequence

$$\underbrace{\sqrt[3]{a + \sqrt[3]{a + \cdots + \sqrt[3]{a}}}}_n$$

is proved, and some inequalities involving this sequence are established for $a > 0$. As by-product, two identities involving irrational numbers are obtained. Two open problems are proposed.

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