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L^p INEQUALITIES FOR ENTIRE FUNCTIONS OF EXPONENTIAL TYPE

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Abstract. Let $f(z)$ be an entire function of exponential type τ and for any complex number ζ, let $D_\zeta[f(z)] = \tau f(z) + i(1-\zeta)f'(z)$ be the polar derivative of $f(z)$, with respect to ζ. This definition is due to Rahman and Schmeisser [10]. Since $\lim_{\zeta\to\infty} \dfrac{D_\zeta[f(z)]}{-i\zeta} = f'(z)$, the polar derivative is a generalization of the ordinary derivative. In this paper we obtain L^p inequalities for the polar derivative of entire functions of exponential type satisfying $f(z) \equiv e^{i\tau z}\{\overline{f(\overline{z})}\}$ and for functions satisfying $f(z) \equiv e^{i\tau z} f(-z)$. Our results generalize some of the known results.

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