

## INEQUALITIES DESCRIBING THE GROWTH OF POLYNOMIALS NOT VANISHING IN A DISK OF PRESCRIBED RADIUS

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*Abstract.* In this paper we study the growth of polynomials of degree  $n$  having no zeros in  $|z| < \kappa$ , where  $\kappa$  is an arbitrary positive number. Using the notation  $M(p; t) = \max_{|z|=t} |p(z)|$  we measure the growth of  $p$  by estimating  $M(p; t)/M(p; 1)$  from above for any  $t > 1$ , and from below for any  $t < 1$ .

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