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REVERSE INEQUALITIES ON CHAOTICALLY GEOMETRIC MEAN VIA SPECHT RATIO

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Abstract. As an application of Mond-Pečarić method, we shall estimate bounds of operator convexity for convex functions. Consequently, we obtain some order relations between the arithmetic mean and the chaotically geometric one $A\diamondsuit_\alpha B$ of positive operators A and B, i.e., $A\diamondsuit_\alpha B = e^{(1-\alpha)\log A+\alpha\log B}$ for $\alpha\in[0,1]$. Among others, we show that if $0<m\leqslant A,\ B\leqslant M$ for some scalars $m<M$ and $h=\frac{M}{m}$, then

$$M_h(1)^{-1}A\diamondsuit_\alpha B\leqslant A\nabla_\alpha B\leqslant M_h(1)A\diamondsuit_\alpha B$$

holds for all $\alpha\in[0,1]$, where the Specht ratio $M_h(1)$ is defined as

$$M_h(1)=\frac{h^{\frac{1}{h-1}}}{e\log h^{\frac{1}{h-1}}}\quad(h>1)\quad\text{and}\quad M_1(1)=1.$$

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