CONVEXITY ACCORDING TO MEANS

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Abstract. Given a function $f : I \to J$ and a pair of means M and N, on the intervals I and J respectively, we say that f is MN-convex provided that $f(M(x,y)) \leqslant N(f(x), f(y))$ for every x, $y \in I$. In this context, we prove the validity of all basic inequalities in Convex Function Theory, such as Jensen's Inequality and the Hermite-Hadamard Inequality.

REFERENCES

[1] H. ALZER, *On an integral inequality*, Math. Rev. Anal. Numer. Th. Approx., **18** (1989), 101–103.

[2] E. ARTIN, *The Gamma Function*, Holt, Rinehart and Winston, New York, 1964. English translation of German original, *Einführung in die Theorie der Gammafunktion*, Teubner, 1931.

[3] H. BOHR AND J. MOLLERUP, *Laerebogi i mathematisk Analyse*, vol. III, pp. 149–164, Kopenhagen, 1922.

[4] D. BORWEIN, J. BORWEIN, G. FEE AND R. GIRGENSOHN, *Refined convexity and special cases of the Blaschke-Santalo inequality*, Math. Inequal. Appl., **4** (2001), 631–638.

[5] J. L. BRENNER AND B. C. CARLSON, *Homogeneous mean values: weights and asymptotics*, J. Math. Anal. Appl., **123** (1987), 265–280.

[6] P. S. BULLEN, D. S. MITRINOVIĆ AND P. M. VASIĆ, *Means and Their Inequalities*, D. Reidel Publishing Company, Dordrecht, 1988.

[7] G. T. CARGO, *Comparable means and generalized convexity*, J. Math. Anal. Appl., **12** (1965), 387–392.

[8] B. C. CARLSON, *Algorithms involving arithmetic and geometric means*, Amer. Math. Monthly, **78** (1971), 496–505.

[9] N. ELEZOVIĆ AND J. PEČARIĆ, *Differential and integral F -means and applications to digamma function*, Math. Inequal. Appl., **3** (2000), 189–196.

[10] G. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities*, Cambridge Mathematical Library, 2nd ed., 1952, Reprinted 1988.

[11] J. LAMPERTI, *On the isometries of certain function-spaces*, Pacific J. Math., **8** (1958), 459–466.

[12] S. M. MALAMUD, *Some complements to the Jensen and Chebyshev inequality and a problem of W. Walter*, Proc. Amer. Math. Soc., **129** (2001), 2671–2678.

[13] J. MATKOWSKI AND J. RÄTZ, *Convexity with respect to an arbitrary mean*, Int. Ser. Numer. Math., **123** (1997), 231–247.

[14] P. MONTEL, *Sur les functions convexes et les fonctions sousharmoniques*, Journal de Math., (9) **7** (1928), 29–60.

[15] M. E. MAYES, *Functions which parametrize means*, Amer. Math. Monthly, **90** (1983), 677–683.

[16] C. P. NICULESCU, *Convexity according to the geometric mean* , Math. Inequal. Appl., **3** (2000), 155–167.

[17] C. P. NICULESCU, *A multiplicative mean value and its applications.* In: *Inequality Theory and Applications* vol. 1, pp. 243–256, Nova Science Publishers (USA), 2001. (Y. J. Cho, S. S. Dragomir and J. Kim, Editors)

[18] A. W. ROBERTS AND D. E. VARBERG, *Convex functions,* Academic Press, New York and London, 1973.

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