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RADICAL AND RATIONAL MEANS OF DEGREE TWO

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Abstract. In this paper, we introduce two families of means that encompass many of the classical means and we characterize the internality properties in these families. We determine the comparability relations within these two families and we study their behavior under equal increments of the variables. We also introduce a geometrical context that gives rise to one of these families.

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