

ON CHAOTIC ORDER OF TWO OPERATORS ON HILBERT SPACE

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Abstract. In this article we characterize an inequality resulted from the chaotic order of two operators on a Hilbert space in terms of Heinz-Kato-Furuta-type inequalities. Consequently, some new characterizations of the weaker version for the Löwner-Heinz inequality are obtained. Finally we introduce the notion of high-order Cauchy-Schwarz inequalities, and its applications are given.

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