Mathematical Inequalities & Applications
Volume 6, Number 4 (2003), 713–714

SIMPLE PROOF OF JOINTLY CONCAVITY OF THE RELATIVE OPERATOR ENTROPY $S(A|B) = A^{\frac{1}{2}} \log(A^{\frac{-1}{2}} B A^{\frac{-1}{2}}) A^{\frac{1}{2}}$

TAKAYUKI FURUTA, MARIKO GIGA AND MASAHIRO YANAGIDA

Abstract. A capital letter means a bounded linear and *strictly positive* operator on a Hilbert space H. Here we shall give a simple proof of the result in [2] [3] that the relative operator entropy $S(A|B) = A^{\frac{1}{2}} \log(A^{\frac{-1}{2}} B A^{\frac{-1}{2}}) A^{\frac{1}{2}}$ is subadditive and jointly concave.

Mathematics subject classification (2000): 47A63.
Key words and phrases: relative operator entropy, subadditive, jointly concave.

REFERENCES

[1] CH. DAVIS, *Operator-valued entropy of a quantum mechanical measurement*, Proc. Japan Acad., **37** (1961), 533–538.
[2] J. I. FUJII AND E. KAMEI, *Relative operator entropy in noncommutative information theory*, Math. Japonica., **34** (1989), 341–348.
[3] J. I. FUJII, M. FUJII AND Y. SEO, *An extension of the Kubo-Ando theory: Solidarities*, Math. Japonica., **35** (1990), 387–396.
[4] T. FURUTA, *Simple proof of the concavity of operator entropy $f(A) = -A \log A$*, Math. Inequal. Appl., **3** (2000), 305–306.
[5] F. KUBO AND T. ANDO, *Means of positive linear operators*, Math. Ann., **246** (1980), 205–224.
[6] M. NAKAMURA AND H. UMEGAKI, *A note on the entropy for operator algebras*, Proc. Japan Acad.,**37** (1961), 149–154.

Paper MIA-06-64