

HYPERMULTITREES AND SHARP BONFERRONI INEQUALITIES

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Abstract. The concept of (h, m) -hypermultitree (which is a special hypergraph) is introduced to present Bonferroni-type inequalities which are equalities for some families of sets. The main theorem is a common generalization of the earlier results of Hunter, Worsley, Tomescu and recent results of Prékopa and the author. The new bounds are based on significantly fewer probabilities than the same order Bonferroni bounds. This and some other properties explain the very high efficiency of the new bounds in many applications, e.g., estimate the values of multivariate normal distribution functions, which is demonstrated in the end of the paper.

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