Mathematical
Inequalities
& Applications
Volume 6, Number 4 (2003), 745–763

OPTIMIZERS FOR SUB–SUMS SUBJECT TO A SUM– AND A SCHUR–CONVEX CONSTRAINT WITH APPLICATIONS TO ESTIMATION OF EIGENVALUES

A. KOVAČEC, J. K. MERIKOSKI, O. PIKHURKO, A. VIRTANEN

Abstract. A complete solution is presented for the problem of determining the sets of points at which the functions $(x_1, \ldots, x_n) \mapsto x_k + \ldots + x_l$, subject to the constraints $M \geqslant x_1 \geqslant \ldots \geqslant x_n \geqslant m$, $x_1 + x_2 + \ldots + x_n = a$, and $g(x_1) + g(x_2) + \ldots + g(x_n) = b$, with g strictly convex continuous, assume their maxima and minima. Applications are given.

Mathematics subject classification (2000): 90C25, 26D15, 15A42.
Key words and phrases: Optimization, Schur-convexity, eigenvalue estimation.

REFERENCES

[1] R. B. BAPAT, *Majorization and singular values III,* Linear Algebra Appl. 145: 59–70 (1991).
[2] M. BERGER, *Geometry I,* Springer, 1987.
[3] J. M. BORWEIN AND A. S. LEWIS, *Convex Analysis and Nonlinear Optimization,* Springer, 2000.
[4] J. M. BORWEIN, G. P. H. STYAN AND H. WOLKOWICZ, *Some inequalities involving statistical expressions: Solution to problem 81–10 of L. V. Foster,* SIAM Review 24: 340–342 (1982). Reprinted in M. S. Klamkin (ed.), *Problems in Applied Mathematics: Selections from SIAM Review,* pp. 373–375, SIAM, 1990.
[5] B. GRONE, C. JOHNSON, E. M. DE SÁ AND H. WOLKOWICZ, *Improving Hadamard's inequality,* Linear and Multilinear Algebra 16: 305–322 (1984).
[6] A. KOVAČEC, Polyhedra of monotone n-tuples; Pré-publicação 01–17, Universidade de Coimbra, Departamento de Matemática, 2001.
[7] K. LEICHTWEISS, *Konvexe Mengen*, Springer, 1980.
[8] A. W. MARSHALL AND I. OLKIN, *Inequalities: Theory of Majorization and its Applications,* Academic Press, 1979.
[9] J. K. MERIKOSKI, H. SARRIA AND P. TARAZAGA, *Bounds for singular values using traces,* Linear Algebra Appl. 210: 227–254 (1994).
[10] J. K. MERIKOSKI, G. P. H. STYAN AND H. WOLKOWICZ, *Bounds for ratios of eigenvalues using traces,* Linear Algebra Appl. 55: 105–124 (1983).
[11] J. K. MERIKOSKI, U. URPALA AND A. VIRTANEN, *Upper bounds for the ratios of the largest and smallest eigenvalues*, Department of Mathematical Sciences, University of Tampere, Report A 307, 1996.
[12] J. K. MERIKOSKI, U. URPALA, A. VIRTANEN, T. Y. TAM AND F. UHLIG, *A best upper bound for the 2-condition number of a matrix,* Linear Algebra Appl. 254: 355–365 (1997).
[13] J. K. MERIKOSKI AND A. VIRTANEN, *Bounds for eigenvalues using the trace and determinant,* Linear Algebra Appl. 264: 101–108 (1997).
[14] J. K. MERIKOSKI AND H. WOLKOWICZ, *Improving eigenvalue bounds using extra bounds,* Linear Algebra Appl. 68: 93–113 (1985).
[15] A. W. ROBERTS AND D. E. VARBERG, *Convex Functions,* Academic Press, 1973.
[16] R. T. ROCKAFELLAR, *Convex Analysis,* Princeton University Press, 1970.
[17] H. WOLKOWICZ AND G. P. H. STYAN, *Bounds for eigenvalues using traces,* Linear Algebra Appl. 29: 471–506 (1980).
[18] H. WOLKOWICZ AND G. P. H. STYAN, *More bounds for eigenvalues using traces,* Linear Algebra Appl. 31: 1–17 (1980).

Paper MIA-06-67