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ON THE GENERALIZED HARDY–HILBERT INEQUALITY AND ITS APPLICATIONS

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Abstract. A generalized Hardy-Hilbert inequality with weight function of the form $B\left(\frac{p-2+\lambda}{p}, \frac{q-2+\lambda}{q}\right) - \theta_r(\lambda)/(2n+1)^{\lambda-\frac{2-\lambda}{r}}$ (with $\theta_r(\lambda) > 0$, $r = p$, q, $1-\frac{q}{p} < \lambda \leqslant 2$, $\frac{1}{p}+\frac{1}{q} = 1$ and $p \geqslant q > 1$) can be established by means of Euler-Maclaurin summation formula, where $B(m,n)$ is β function. In particular, when $\lambda = 1$, an improvement on Hardy-Hilbert's inequality is obtained. As its applications, Hardy-Littlewood's inequality is extended and refined.

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