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NECESSARY AND SUFFICIENT TAUBERIAN CONDITIONS IN THE CASE OF WEIGHTED MEAN SUMMABLE INTEGRALS OVER $\mathbb{R}_+$

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Abstract. Let $0 \not\equiv p(x)$ be a nondecreasing function on $\mathbb{R}_+ := [0,\infty)$ such that $p(0) = 0$ and

$$\liminf_{t\to\infty} p(\lambda t)/p(t) > 1 \quad \text{for every} \quad \lambda > 1.$$

Given a real- or complex-valued function $f \in L^1_{\mathrm{loc}}(\mathbb{R}_+)$, we define

$$s(x) := \int_0^x f(u)du \quad \text{and} \quad \sigma(t) := \frac{1}{p(t)}\int_0^t s(x)dp(x), \quad t > 0.$$

It is known that if the finite limit $\lim_{x\to\infty} s(x) = L$ exists, then the limit $\lim_{t\to\infty} \sigma(t) = L$ also exists. Our goal is to find necessary and sufficient conditions under which the converse implication holds. Most of these conditions are expressed in terms of inequalities.

In the case of real-valued functions we present one-sided Tauberian conditions, while in the case of complex-valued functions we present two-sided Tauberian conditions. As special cases, we obtain well-known Tauberian conditions such as slow decrease in the sense of R. Schmidt, slow oscillation in the sense of Hardy, and Landau type Tauberian c onditions.

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