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OPERATOR MONOTONE FUNCTIONS INDUCED FROM LÖWNER–HEINZ INEQUALITY AND STRICTLY CHAOTIC ORDER

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Abstract. Furuta presented direct and simplified proofs of operator monotonicity of functions

$$\varphi(t) = \frac{t-1}{\log t} \quad \text{and} \quad \psi(t) = \frac{t\log t - t + 1}{(\log t)^2}$$

by using Löwner-Heinz inequality. Extending his method, we give a sequence of operator monotone functions $\{f_k(t)\}_{k=0}^{\infty}$ with $f_0(t) = \varphi(t)$ and $f_1(t) = \psi(t)$. We also study relations between $f_k(t)$ and strictly chaotic order defined among positive invertible operators and obtain some extensions of results due to Furuta.

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