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SPECHT'S RATIO AND LOGARITHMIC MEAN IN THE YOUNG INEQUALITY

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Abstract. For a positive operator A with $0 < m \leqslant A \leqslant M$ $(m, M \in \mathbb{R})$, the Young operator inequality gives as follows: $\lambda A + (1-\lambda) \geqslant A^{\lambda}$ for $\lambda \in [0,1]$.

In this note, we prove that the estimation of the converse Young operator inequality is obtained by using Specht's ratio $S(t) = \dfrac{t^{\frac{1}{t-1}}}{e \log t^{\frac{1}{t-1}}}$ and the logarithmic mean $L(s,t) = \frac{t-s}{\log t - \log s}$ $(s,t>0)$, that is, we have for a given p under some conditions

$$pA^{\lambda} + \max\left\{ L(1,m) \log \frac{S(m)}{p}, L(1,M) \log \frac{S(M)}{p} \right\} \geqslant \lambda A + (1-\lambda)\ (\geqslant A^{\lambda}) \quad \text{for } \lambda \in [0,1].$$

Moreover by using operator means, we consider the converse Young operator inequality related to two operators A and B.

Furthermore we discuss reverse inequalities of the Hölder-McCarthy inequality and the inequality on the concavity of the logarithmic function.

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