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A GENERAL FRAMEWORK FOR THE SOLVABILITY OF A CLASS OF NONLINEAR VARIATIONAL INEQUALITIES

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Abstract. Based on a general framework for the auxiliary problem principle involving continuously m-Frechet-differentiable $(m \geqslant 2)$ mappings, the approximation-solvability of the following class of nonlinear variational inequality problems (NVIP) involving the generalized partially relaxed monotone mappings is presented.

Find an element $x^* \in K$ such that

$$\langle T(x^*), \eta(x, x^*)\rangle + f(x) - f(x^*) \geqslant 0 \quad \text{for all} \quad x \in K,$$

where $T : K \to \mathbf{R}^n$ is a mapping from a nonempty closed invex subset K of $\mathbf{R}^n$ into $\mathbf{R}^n$, $\eta : K \times K \to \mathbf{R}^n$ is a mapping, and $f : K \to \mathbf{R}$ is a continuous invex function on K. The general class of the auxiliary problems principle is described as follows: for a given iterate $x^k \in K$ and for a parameter $\rho > 0$, determine x^{k+1} such that

$$\langle \rho T(x^k) + h'(x^{k+1}) - h'(x^k), \eta(x, x^{k+1})\rangle + \rho[f(x) - f(x^{k+1})] \geqslant 0 \quad \text{for all} \quad x \in K,$$

where $h : K \to R$ is continuously Frechet-differentiable on K.

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