BERNSTEIN–DOETSCH TYPE RESULTS
FOR QUASICONVEX FUNCTIONS

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Abstract. In this paper various quasiconvexity notions are considered and compared. The main goal is to show that, under the assumption of upper semicontinuity, Jensen-type quasiconvexity properties are equivalent to the corresponding ordinary quasiconvexity property. The results thus obtained are analogous to the classical theorem of Bernstein and Doetsch for convex functions.

Finally, the connection between approximate Jensen quasiconvexity and approximate quasiconvexity is investigated.

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