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MAXIMAL FUNCTION ON GENERALIZED LEBESGUE SPACES $L^{p(\cdot)}$

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Abstract. We prove the boundedness of the Hardy–Littlewood maximal function on the generalized Lebesgue space $L^{p(\cdot)}(\mathbb{R}^d)$ under a continuity assumption on p that is weaker than uniform Hölder continuity. We deduce continuity of mollifying sequences and density of $C^\infty(\overline{\Omega})$ in $W^{1,p(\cdot)}(\Omega)$.

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