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HARDY–LITTLEWOOD MAXIMAL OPERATOR ON $L^{p(x)}(\mathbb{R})$

ALEŠ NEKVINDA

Abstract. We consider Hardy-Littlewood maximal operator on the general Lebesgue space $L^{p(x)}(\mathbb{R}^n)$ with variable exponent. A sufficient condition on the function p is known for the boundedness of the maximal operator on $L^{p(x)}(\Omega)$ with an open bounded Ω. Our main aim is to find an additional condition to p to guarantee the boundedness of the maximal operator on $L^{p(x)}(\mathbb{R}^n)$. From this point of view we put an emphasis on the behavior of functions p near the infinity. We find a sufficient condition on p such that the maximal operator is bounded on $L^{p(x)}(\mathbb{R}^n)$. We also construct a function p for which the maximal operator is unbounded.

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