Mathematical
Inequalities
& Applications
Volume 7, Number 3 (2004), 309–313

A VARIATIONAL PROOF OF BIRKHOFF'S THEOREM ON DOUBLY STOCHASTIC MATRICES

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Abstract. This note provides a short variational proof of the Birkhoff's theorem asserting that the extreme points of the convex set of doubly stochastic matrices are the permutation matrices.

Mathematics subject classification (2000): 15A42, 15A51, 58E30.
Key words and phrases: Variational method, doubly stochastic matrix, majorization, Birkhoff theorem.

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Paper MIA-7-34