

## OPIAL'S INEQUALITY FOR ZERO-AREA CONSTRAINT

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*Abstract.* We prove that  $\int_0^1 |yy'| \leq \frac{1}{4} \int_0^1 y^2$  provided  $\int_0^1 y = 0$ . The nontrivial part is the sharp constant  $\frac{1}{4}$ . Cases of equality can be abstracted from the proof. This is an (apparently much deeper) variant of a result by Opial, who proved the same estimate under Dirichlet boundary conditions on  $y$ .

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