Mathematical
Inequalities
& Applications
Volume 7, Number 3 (2004), 379–391

INEQUALITIES FOR A POLYNOMIAL AND ITS DERIVATIVE

A. AZIZ AND W. M. SHAH

Abstract. In this paper we consider a class of polynomials $P(z) = a_0 + \sum_{j=\mu}^{n} a_j z^j$, $1 \leqslant \mu \leqslant n$, not vanishing in the disk $|z| < K$. For $K \geqslant 1$, we investigate the dependence of $\max_{|z|=1} |P(Rz) - P(z)|$ on $\max_{|z|=1} |P(z)|$ and for $K > 0$ we estimate $\max_{|z|=R} |P'(z)|$ in terms of $\max_{|z|=r} |P(z)|$, $0 \leqslant r \leqslant R \leqslant K$. Our results not only generalize some known polynomial inequalities, but olso a variety of interesting results can be deduced from these by a fairly uniform procedure. We also obtain a generalization of a Theorem of Paul Turan.

Mathematics subject classification (2000): 30A10, 30C10, 30D15.
Key words and phrases: Polynomials, inequalities.

REFERENCES

[1] N. C. ANKENY AND T. J. RIVLIN, *On a theorem of S. Bernstein*, Pacific J. Math. **5** (1955), 849–852.
[2] A. AZIZ, *Inequality for the derivative of a polynomial*, Proc. Amer. Math. Soc. **89** (1983), 259–266.
[3] A. AZIZ AND Q. M. DAWOOD, *Inequalities for a polynomial and its deivative*, J. Approx. Theory. **54** (1988), 306–313.
[4] A. AZIZ AND N. A. RATHER, *New L^q inequalities for polynomials*, Mathematical Inequalities and Applications (1999), 177–191.
[5] A. AZIZ AND W. M. SHAH, *L^q inequlities for polynomials with restricted zeros*, Glasnik Matematički **32** (1997), 249–258.
[6] S. BERNSTEIN, *Lecons Sur Les Proprie'le's extre'males et la meilleure approximation des fonctions analytiques d'une fonctions reele*, Paris, 1926.
[7] T. N. CHAN AND M. A. MALIK, *On Erdös-Lax theorem*, Proc. Indian Acad. Sci. **137** (1969), 501–517.
[8] B. DATT, *Ph. D. Thesis*, submitted to the Indian Institute of Technology, Delhi, 1976.
[9] K. K. DEWAN, *Ph. D. Thesis*, submitted to the Indian Institute of Technology, Delhi, 1980.
[10] K. K. DEWAN AND M. BIDKHAM, *Inequalities for a polynomial and its derivative*, J. Math. Anal. Appl. **166** (1992), 319–324.
[11] C. FRAPPIER, Q. I. REHMAN AND S. T. RUSCHEWEYH, *New inequalities for polynomials*, Trans. Amer. Math. Soc. **288** (1985), 69–99.
[12] N. K. GOVIL, Q. I. RAHMAN AND G. SCHMEISSER, *On the derivative of a polynomial*, Illinois J. Math. **23** (1979), 319–329.
[13] P. D. LAX, *Proof of a conjecture of P. Erdös on the derivative of polynomial*, Bull. Amer. Math. Soc. (N. S.) **50** (1944), 509–513.
[14] M. A. MALIK, *On the derivative of a polynomial*, J. London Math. Soc. **1** (1969), 57–60.
[15] G. V. MILOVANOVIĆ, D. S. MITRINOVIĆ AND TH. M. RASSIAS, *Topics in polynomials, Extremal Problems, Inequalities, zeros*, World Scientific, Singapore, 1994.
[16] M. S. PUKHTA, *Ph. D. thesis*, submitted to the Jamia Millia Islamia, New Delhi, 1995.
[17] M. A. QAZI, *On the maximum modulus of polynomials*, Proc. Amer. Math. Soc. **115** (1992), 337–343.
[18] M. RIEZE, *Über einen Satz der Hern Serge Bernstein*, Acta Math. **40** (1918), 337–347.
[19] P. TURA'N, *Über die ablaeitung von polynomen*, Compositio Math. Soc. **25** (1993), 49–54.

© ELEMENT, Zagreb
Paper MIA-07-39