

SOME COMPACT GENERALIZATIONS OF BERNSTEIN-TYPE INEQUALITIES FOR POLYNOMIALS

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Abstract. Let $P(z)$ be a polynomial of degree $n \geq 1$. In this paper we consider a more general problem of investigating the dependence of maximum of

$$\left| P(Rz) - \alpha P(z) + \beta \left\{ \left(\frac{R+1}{2} \right)^n - |\alpha| \right\} P(z) \right|, \quad R \geq 1,$$

on the maximum of $|P(z)|$ on $|z| = 1$ where α, β are arbitrary complex numbers with $|\alpha| \leq 1, |\beta| \leq 1$ and obtain certain sharp compact generalizations of well-known Bernstein-type polynomial inequalities.

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