

FUNCTIONAL INCLUSIONS ON SQUARE-SYMMETRIC GRUPOIDS AND HYERS-ULAM STABILITY

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Abstract. In this paper we prove that a set-valued map $F : X \rightarrow P_0(Y)$ that satisfies the inclusion $F(x * y) \subset F(x) \diamond F(y)$ under suitable conditions admits exactly one selection $f : X \rightarrow Y$ that satisfies the equation $f(x * y) = f(x) \diamond f(y)$, where $(X, *)$ and (Y, \diamond) are square-symmetric grupoids and \diamond is the extension of \diamond to $P_0(Y)$. This result is in connection with Hyers-Ulam stability of functional equation and generalizes a result of Z. Gajda and R. Ger.

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