UNIFORM NON–SQUARENESS OF ψ –DIRECT
SUMS OF BANACH SPACES $X \oplus_\psi Y$

Mikio Kato, Kichi-Suke Saito and Takayuki Tamura

Abstract. We characterize the uniform non-squareness of the ψ -direct sum $X \oplus_\psi Y$ of Banach spaces X and Y, where ψ is a convex function on the unit interval satisfying certain conditions. As a corollary the uniform non-squareness of an $\ell_{p,q}$ -sum $X \oplus_{p,q} Y$ is characterized. In the course of doing this a monotonicity property of absolute norms on $\mathbb{C}^2$ is shown.

REFERENCES

[1] B. Beauzamy, *Introduction to Banach Spaces and their Geometry*, 2nd ed., North-Holland, 1985.
[2] F. F. Bonsall and J. Duncan, *Numerical Ranges II*, London Math. Soc. Lecture Note Ser. **10** (1973).
[3] R. Bhatia, *Matrix Analysis*, Springer, 1997.
[4] G. H. Hardy, J. E. Littlewood and G. Pólya, *Inequalities*, Cambridge Univ. Press, 1967.
[5] C. James, *Uniformly non-square Banach spaces*, Ann. of Math. **80** (1964), 542–550.
[6] M. Kato, *On Lorentz spaces $\ell_{p,q}\{E\}$*, Hiroshima Math. J. **6** (1976), 73–93.
[7] M. Kato, K.-S. Saito and T. Tamura, *On the ψ -direct sums of Banach spaces and convexity*, J. Aust. Math. Soc. **75** (2003), 413–422.
[8] R. E. Megginson, *An Introduction to Banach Space Theory*, Springer, 1998.
[9] K. Mitani, K.-S. Saito and T. Suzuki, *Smoothness of absolute norms on $\mathbb{C}^n$*, J. Convex Anal. **10** (2003), 89–107.
[10] K.-S. Saito and M. Kato, *Uniform convexity of ψ -direct sums of Banach spaces*, J. Math. Anal. Appl. **277** (2003), 1–11.
[11] K.-S. Saito, M. Kato and Y. Takahashi, *Von Neumann-Jordan constant of absolute normalized norms on $\mathbb{C}^2$*, J. Math. Anal. Appl. **244** (2000), 515–532.
[12] K.-S. Saito, M. Kato and Y. Takahashi, *On absolute norms on $\mathbb{C}^n$*, J. Math. Anal. Appl. **252** (2000), 879–905.
[13] Y. Takahashi, M. Kato and K.-S. Saito, *Strict convexity of absolute norms on $\mathbb{C}^2$ and direct sums of Banach spaces*, J. Inequal. Appl. **7** (2002), 179–186.
[14] H. Triebel, *Intepolation Theory, Function spaces, Differential Operators*, North-Holland, 1978.

ⓒ ELEMENT, Zagreb
Paper MIA-07-43

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com