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THE FIRST WEIERSTRASS–ERDMANN CONDITION IN VARIATIONAL PROBLEMS INVOLVING DIFFERENTIAL INCLUSIONS

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Abstract. In this paper, the authors continue a previous study about the broken extremals in variational problems with differential inclusions. In said paper, we presented a necessary condition for extremals with corner points that is valid for shapeable sets. This condition has been obtained by adapting a novel proof of the first Weiertrass-Erdmann condition.

In the present paper we extend the class of shapeable sets and demonstrate that the set

$$\Omega := \{z \in KC^1[a,b] \mid G_1(t,z(t)) \leqslant z'(t) \leqslant G_2(t,z(t)), \forall t \in [a,b] \text{ a.e.}\}$$

with $G_1, G_2 \in C^1$, is shapeable for every t.

Finally, we present two examples, the second being a classic engineering problem: the optimization of hydrothermal systems.

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REFERENCES

[1] L. BAYÓN, J. M. GRAU, P. M. SUÁREZ, *A Necessary Condition for Broken Extremals in Problems Involving Inequality Constraints*, Archives of Inequalities and Applications, Vol. **1**(1), (2003), 75–84.
[2] L. BAYÓN, J. M. GRAU, P. M. SUÁREZ, *A New Formulation of the Equivalent Thermal in Optimization of Hydrothermal Systems*, Math. Probl. Eng., Vol. **8**(3), (2002), 181–196.
[3] L. BAYÓN, J. M. GRAU, M. M. RUIZ, P. M. SUÁREZ, *A Optimization technique of Hydrothermal Systems using Calculus of Variations*, Proceedings ICREPQ 2003, Spain, 2003.
[4] M. CESAR, *Reformulation of the second Weierstrass-Erdmann condition*, Bol. Soc. Brasil. Mat. **13**(1982), no. 1, 19–23.
[5] M. CESAR, *Necessary conditions and sufficient conditions of weak minimum for solutions with corner points*, Bol. Soc. Brasil. Mat, **15**(1984), no. 1–2, 109–135.
[6] L. CESARI, *Optimization — theory and applications. Problems with ordinary differential equations*, Applications of Mathematics, **17**, Springer-Verlag, New York, 1983.
[7] F. H. CLARKE, *Inequality constraints in the calculus of variations*, Canad. J. Math. **29**, no. 3 (1977), 528–540.
[8] F. H. CLARKE, *The Erdmann condition and Hamiltonian inclusions in optimal control and the calculus of variations*, Canad. J. Math. **32**, no. 2 (1980), 494–509.
[9] F. H. CLARKE AND P. D. LOEWEN, *State constraints in optimal control: a case study in proximal normal analysis*, SIAM J. Control Optim. **25**, no. 6 (1987), 1440–1456.
[10] F. H. CLARKE AND P. D. LOEWEN, *An intermediate existence theory in the calculus of variations*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. **16**, no. 4 (1990), 487–526.
[11] M. E. EL-HAWARY AND G. S. CHRISTENSEN, *Optimal economic operation of electric power systems*, Academic Press, New York, 1979.
[12] G. ERDMANN, *Journal für die Reine und Angewandte Mathematik*, **82**(1875), 21–30.
[13] B. FLODIN, *Über diskontinuierliche Lösungen bei Variationsproblemen mit Gefällbeschränkung*, Acta Soc. Sci. Fennicae. Nova Ser. A. **3**, no. 10 (1945), 31 pp.

[14] O. FÖLLINGER, *Diskontinuierliche Lösungen von Variationsproblemen mit Gefällbeschränkung*, Math. Annalen, Vol. **126**(1953), 466–480.
[15] I. M. GELFAND, S. V. FOMIN, *Calculus of Variations*, Prentice-Hall, 1963.
[16] M. GIAQUINTA, S. HILDEBRANDT, *Calculus of Variations*, Springer-Verlag, New York, 1996.
[17] P. D. LOEWEN AND R. T. ROCKAFELLAR, *New necessary conditions for the generalized problem of Bolza*, SIAM J. Control Optim. **34**, no. 5 (1996), 1496–1511.
[18] P. D. LOEWEN AND R. T. ROCKAFELLAR, *Bolza problems with general time constraints*, SIAM J. Control Optim. **35**, no. 6 (1997), 2050–2069.
[19] J. NOBLE, H. SCHÄTTLER, *On sufficient conditions for a strong local minimum of broken extremals in optimal control*, Res. Notes Math., **396**(1999), 171–179.
[20] J. NOBLE, H. SCHÄTTLER, *Sufficient conditions for relative minima of broken extremals in optimal control theory*, J. Math. Anal. Appl., **269** (2002), 98–128.
[21] N. P. OSMOLOVSKII, F. LEMPIO, *Jacobi conditions and the Riccati equation for a broken extremal*, Pontryagin Conference, 1, Optimal Control (Moscow, 1998). J. Math. Sci. (New York) **100** (2000), no. 5, 2572–2592.
[22] N. P. OSMOLOVSKII, *Second-order conditions for broken extremal*, Calculus of variations and optimal control (Haifa, 1998), 198–216, Chapman & Hall/CRC Res., Notes Math., 411, Chapman & Hall/CRC, Boca Raton, FL, 2000.
[23] N. P. OSMOLOVSKII, F. LEMPIO, *Transformation of Quadratic Forms to Perfect Squares for Broken Extremals*, Set-Valued Analysis **10**(2) (2002), 209–232.
[24] L. A. PARS, *An introduction to the calculus of variations*, John Wiley & Sons Inc., New York, 1962.
[25] M. A. SYTCHEV, *Qualitative properties of solutions of the Euler equation and the solvability of one-dimensional regular variational problems in the classical sense*, Siberian Math. J. **36** (1995), no. 4, 753–769.
[26] J. L. TROUTMAN, *Variational calculus with elementary convexity*, Springer-Verlag, New York, 1983.