

LORENTZ SPACES FOR DECREASING REARRANGEMENTS OF FUNCTIONS ON TREES

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Abstract. We characterize the existence of a norm in the Lorentz spaces defined on trees, for non-linear decreasing rearrangements, as well as other functional properties (quasinormability, relationship with rearrangement invariant spaces, etc.). The main tool is the characterization of the saturation in Hardy-Littlewood's inequality for linearly decreasing functions.

Mathematics subject classification (2000): 05C05, 46E30, 46B25.

Key words and phrases: homogeneous trees, decreasing rearrangements, Lorentz spaces.

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