

A NOTE ON BOUNDS FOR NORMS OF THE RECIPROCAL LCM MATRIX

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Abstract. Let $S = \{x_1, x_2, \dots, x_n\}$ be a set of distinct positive integers and $[x_i, x_j]$ denote the least common multiple of x_i and x_j . The matrix $[S^{-1}] = (s_{ij})$, where $s_{ij} = \frac{1}{[x_i, x_j]}$, is called the reciprocal least common multiple (reciprocal LCM) matrix on S . In this paper, we investigate some matrix norms of the reciprocal LCM matrix and one of its generalizations on $S = \{1, 2, \dots, n\}$ in terms of the Riemann zeta function.

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