

A CONJECTURE ABOUT THE INERTIA OF HERMITIAN MATRICES

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Abstract. Let H be a Hermitian matrix which has been decomposed into m rows and m columns of blocks. Suppose further that we know the inertia of each diagonal block and a range of possible ranks for each off-diagonal block. What are the possible inertias of H ?

In this paper, a conjecture on the inertias of Hermitian matrices with a prescribed 3×3 block decomposition is presented, based on several important works on the subject.

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