LIONS–PEETRE TYPE COMPACTNESS
RESULTS FOR SEVERAL BANACH SPACES

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Abstract. Working with interpolation methods associated to polygons, a result of Cobos and Peetre guarantees that the interpolated operator is compact provided all but two restrictions of the operator (located in adjacent vertices) are compact. We characterize here those intermediate spaces that satisfy the conclusion of Cobos-Peetre result for all operators. We also establish some results on rank-one interpolation spaces.

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