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LOCAL GROWTH ENVELOPES OF SPACES OF GENERALIZED SMOOTHNESS: THE CRITICAL CASE

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Abstract. The concept of local growth envelope of a quasi-normed function space is applied to the spaces of Besov and Triebel-Lizorkin type of generalized smoothness (s, Ψ) in the critical case $s = n/p$, where s stands for the main smoothness, Ψ is a perturbation and p stands for integrability. The expression obtained for the behaviour of the local growth envelope functions (which, as expected, depends on Ψ) shows the ability to be generalized to a form unifying both critical ($s = n/p$) and subcritical ($s < n/p$) cases.

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